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OLYMPIAD PROBLEMS FROM ALL OVER THE WORLD

VOLUME 5
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CR
Cartea Românească
EDUCAȚIONAL

Dedicated to the International Mathematical Olympiad

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Chapter I
Problems

1. Let α be a fixed real number. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(f(x + y)\mathrm{f}(x - y)) = x^2 + \alpha y f(y)$ for all $x, y \in \mathbb{R}$.

WALTER JANOUS, AUSTRIAN NMO, 2017

2. Let M be a set of 2017 positive integers. For every non-empty $A \subset M$, we define $f(A) = \{x \in M : x$ is divisible by odd number of elements of $A\}$.

Find the minimum number of colors such that it is possible to paint all non-empty subset of M in such a way that, whenever $A \neq f(A)$, the sets A and $f(A)$ are in different colors.

ALEKSANDAR IVANOV, BULGARIAN NMO, 2017

3. Let n be a positive integer and $a_1, a_2, ..., a_{2n}$ be $2n$ distinct integers. Given that the equation $|x - a_1|\,|x - a_2| \ldots |x - a_{2n}| = (n!)^2$ has an integer solution $x = m$, find m in terms of $a_1, ..., a_{2n}$.

SINGAPORE, SMO, 2017

4. We consider the real numbers x, y and z, which satisfy the system of equations:

$$\begin{cases} x + y + z = 3 \\ xy + yz + xz = 2 \end{cases}.$$

Compute $\max(x) + \min(x)$.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

5. Given 7 distinct positive integers, prove that there is an infinite arithmetic progression of positive integers $a, a + d, a + 2d, ...$, with $a \leq d$, that contains exactly 3 or 4 of the 7 given integers.

SINGAPORE, SMO, 2017

6. Solve the equation $x^2(2 - x)^2 = 1 + 2(1 - x)^2$.

FINBAR HOLLAND, IRELAND SHL, 2017

7. Show that, for all x, y, z, w, $(x - w)(y - z) + (y - w)(z - x) + (z - w)(x - y) = 0$ and $\sin(x - w)\sin(y - z) + \sin(y - w)\sin(z - x) + \sin(z - w)\sin(x - y) = 0$.

FINBAR HOLLAND, IRELAND SHL, 2017

8. Let $f(n) = 4n^2 + 7n^2 + 3n + 6$. Prove that if n is an integer, then $f(n)$ is not the cube of an integer.

TOM LAFFEY, IRELAND SHL, 2017

9. For each positive integer n, let $c_p = 2017^n$. Suppose that a function $f: \mathbb{N} \to \mathbb{R}$ satisfies the following two conditions:

(i) $f(m + n) \le 2017 \cdot f(m) \cdot f(n + 325)$ for each positive integer m, n;

(ii) $0 < f(c_n + 1) < f(c_n)^{2017}$ for each positive integer n.

Show that there exist a sequence $a_1, a_2, \ldots$, satisfying the following condition.

For all positive integers n, k with $a_k < n$, we have $f(n)^{c_k} < f(c_k)^n$.

In the problem, $\mathbb{N}$ is the set of positive integers and $\mathbb{R}$ is the set of reels.

KOREAN NMO, 2017

10. A function $f: \mathbb{Z} \to \mathbb{Z}$ is called loggy if it satisfies the following two conditions:

(i) $f(xy) \equiv f(x) + f(y) \pmod 8$ for all $x, y \in \mathbb{Z}$ that are not divisible by 17;

(ii) $f(x + 17) \equiv f(x) \pmod 8$ for all $x \in \mathbb{Z}$.

Determine, with proof:

a) if there exists a loggy function for which $f(2) = 1$;

b) if there exists a loggy function for which $f(3) = 1$.

BERND KREUSSLER, IRELAND NMO, 2017

11. Let n be a positive integer and $a_1, \ldots, a_n$ positive real numbers. Let:

$$s_k = a_1^k + \ldots + a_n^k \text{ for } k = 1, 2, 3, \ldots .$$

Prove that $\dfrac{s_5 s_1^3}{5} - \dfrac{s_4 s_2 s_1^2}{4} + \dfrac{s_2^4}{20} \ge 0$.

TOM LAFFEY, IRELAND SHL, 2017

12. Suppose x, y, z are positive numbers that sum to π. Prove that:

$$\frac{\sin 2x + \sin 2y + \sin 2z}{\sin x + \sin y + \sin z} \le 1,$$

with equality if $x = y = z = \dfrac{\pi}{3}$.

FINBAR HOLLAND, IRELAND SHL, 2017

13. There are some boys and some girls at a party. A set of boys is said to be *sociable* if every girl at the party knows at least one boy in that set, and similarly a set of girls is said to be *sociable* if every boy at the party knows at least one girl in that set.

Suppose that the number of sociable sets of boys is odd. Prove that the number of sociable sets of girls is also odd.

Note: Acquaintance is mutual.

MARK FLANAGAN, IRELAND NMO, 2017

14. Suppose A, B, and C are the angles in an acute-angled triangle. Prove that:

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\cos A+\cos B+\cos C}\le\sqrt{3}.$$

FINBAR HOLLAND, IRELAND SHL, 2017

15. In ΔABC, the following relationship holds:

$$ab^7+bc^7+ca^7\ge 62208r^8.$$

DANIEL SITARU, RMM, ROMANIA

16. In ΔABC, the following relationship holds:

$$\prod(m_a+r_a)\ge 8\sqrt{\prod w_a h_a}+\left(\sqrt{\prod m_a}-\sqrt{\prod r_a}\right)^2.$$

DANIEL SITARU, RMM, ROMANIA

17. In ΔABC, the following relationship holds:

$$\left(\sum\sqrt{m_a}\right)^2\ge\sum m_a+6\sqrt[3]{rs^2}.$$

DANIEL SITARU, RMM, ROMANIA

18. If in ΔABC, K-Lemoine's point, then the following relationship holds:

$$\sum\sqrt[3]{a}\cdot KA^2\ge\frac{\sqrt[3]{abc}\left(a\sqrt[3]{a^2}+b\sqrt[3]{b^2}+c\sqrt[3]{c^2}\right)}{\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}}.$$

DANIEL SITARU, RMM, ROMANIA

19. If $a, b, c\ge 0$, then:

$$(\cos 50^\circ+\cos 70^\circ)\sum a^2\ge\frac{2\cos 10^\circ}{1+2\sqrt{3}\cos 10^\circ}\sum(a^2+ab).$$

DANIEL SITARU, RMM, ROMANIA

20. In ΔABC, the following relationship holds:

$$a^2r_a+b^2r_b+c^2r_c\ge 108r^3.$$

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21. In any triangle ABC, the following relationship holds:

$$\sum a^2c^2\sin 2B+\sum b^2c^2\sin 2A\ge 432\sqrt{3}r^4.$$

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22. In ΔABC, the following relationship holds:

$$(a\cot 20^\circ + b\cot 40^\circ + c\cot 80^\circ)^3 > 9\sqrt{3}\left(\frac{a^3}{r_a}+\frac{b^3}{r_b}+\frac{c^3}{r_c}\right).$$

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23. If $a, b \in \mathbb{N} \setminus \{0\}$, then:

$$\left(a+\sqrt{ab}+b\right)\left(\frac{1}{a}+\frac{1}{\sqrt{ab}}+\frac{1}{b}\right)^{ab} \le \left(\frac{3a+3b}{1+ab}\right)^{1+ab}.$$

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24. If $a, b, x, y, z > 0$, then:

$$\sqrt[3]{\left(a+\frac{b(x+y+z)}{x}\right)\left(a+\frac{b(x+y+z)}{y}\right)\left(a+\frac{c(x+y+z)}{z}\right)} \ge a+3b.$$

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25. If in ΔABC $2Rr = 1$, then:

$$16R^2 + 12r^2 \ge 19.$$

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26. If in ΔABC $\tan\frac{B}{2}\tan\frac{C}{2}=\frac{1}{3}$, then:

$$s^2 \ge 27R^2 \sin^2 \frac{A}{2}.$$

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27. If $a, b, c > 0$, then:

$$\left(6\sqrt[3]{\frac{a}{b}}-\frac{a^2}{b^2}\right)+\left(6\sqrt[3]{\frac{b}{c}}-\frac{b^2}{c^2}\right)+\left(6\sqrt[3]{\frac{c}{a}}-\frac{c^2}{a^2}\right) \le 15.$$

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28. In ΔABC, the following relationship holds:

$$\sum \frac{1}{(a+2\sqrt{ab})(b+2\sqrt{ab})} \le \frac{1}{18Rr}.$$

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29. If in ΔABC:

$$x=\cos^{-1}\left(\frac{a}{b+c}\right),\ y=\cos^{-1}\left(\frac{b}{c+a}\right),\ z=\cos^{-1}\left(\frac{c}{a+b}\right),\ \text{then:}$$

$$\tan\frac{x}{2}\tan\frac{y}{2}\tan\frac{z}{2}\le\frac{\sqrt{3}}{9}.$$

DANIEL SITARU, RMM, ROMANIA

30. In ΔABC, the following relationship holds:

$$(a^2+b^2+c^2)\sqrt{a^2+b^2+c^2}\ge 6abc\sqrt{6\cos A\cos B\cos C}.$$

DANIEL SITARU, RMM, ROMANIA

31. Let be $n\in\mathbb{N}^*\setminus\{1\}$ și $a_k\in\mathbb{R}$, $k\in\overline{1,n}$. Prove that:

$$\sum_{k=1}^{n}\sqrt{a_k^2-a_ka_{k+1}+a_{k+1}^2}\ge\sum_{k=1}^{n}a_k;\ a_{n+1}=a_1.$$

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU,
ROMANIA, RMM SUMMER EDITION, 2016

32. Prove that if $a, b, c, d>0$, then:

$$a^2+b^2+c^2+d^2=1;\ abc+bcd+cda+dab=\frac{1}{2}.$$

$$\sum\frac{a^2}{1+2bcd}\ge\frac{4}{5}.$$

DANIEL SITARU, ROMANIA, RMM SUMMER EDITION, 2016

33. If $x, y, z\in(0,\infty)$, then: $x+y+z+\dfrac{1}{x^3}+\dfrac{1}{y^3}+\dfrac{1}{z^3}\ge\dfrac{12}{\sqrt{3\sqrt{3}}}$.

DANIEL SITARU, ROMANIA, RMM SUMMER EDITION, 2016

34. We consider $f(x)=x^3+bx^2+cx+d$ with $f(2014)=2013$ and $g(x)=x^2-2x+2014$ such that the equation $f(g(x))=0$ doesn't have real roots. Solve the equation $f(x)=0$ knowing that it has three distinct natural numbers roots.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

35. If $a, b, c>0$, $(a+b)(b+c)(c+a)=a^2b^2c^2$, then:

$$\sqrt{a+b}+\sqrt{b+c}+\sqrt{c+a}\ge 6.$$

DANIEL SITARU, RMM, ROMANIA

36. If $a, b, c > 0$, $a^2 + b^2 + c^2 = 3$, then:

$$6\left(\frac{b^2}{\sqrt{a^2+3}}+\frac{c^2}{\sqrt{b^2+3}}+\frac{a^2}{\sqrt{c^2+3}}\right)\geq (a+b+c)^2.$$

DANIEL SITARU, RMM, ROMANIA

37. In ΔABC, the following relationship holds:

$$\frac{a^4}{\tan 22^\circ}+\frac{b^2}{\tan 22^\circ \tan 23^\circ}+\frac{c^4}{\tan 23^\circ}>48S^2.$$

DANIEL SITARU, RMM, ROMANIA

38. There are 12 chairs which are aligned and labeled by numbers 1; 2; ...; 12 from left to right. A grasshopper can jump from one chair to another following the rule: from a chair with number k it can jump to the chair with number n if and only if $|k-n| = 5$ or $|k-n| = 8$. It is known that grasshopper managed to do the jump so that it visited all chairs exactly once. What chair could be the initial position for the grasshopper?

UKRAINIAN NMO, 2016

39. Let x, y, z be real numbers from segment $[0; 1]$. Prove that:

$$(x^4+y^4+z^4)+(x^5+y^5+z^5)+(x-y)^6+(y-z)^6+(z-x)^6\leq 6.$$

YASINSKII VYACHESLAV, UKRAINIAN NMO, 2016

40. Find all real numbers x satisfying the following equation:

$$(x+\{x\})^2-(x+\{x\})=6[x]\{x\}-1,$$

where $[x]$ and $\{x\}$ denote the integer part and fractional part of x, respectively.

NGUYEN VIET HUNG, VIETNAM, RMM AUTUMN EDITION, 2016

41. A convex quadrilateral $ABCD$ is inscribed in a circle. The lines AD and BC meet at point E. Points M and N are taken on the sides AD and BC, respectively, so that $AM : MD = BN : NC$. Let the circumcircles of triangle EMN and quadrilateral $ABCD$ intersect at point X and Y. Prove that either the lines AB, CD and XY have a common point, or they are all parallel.

DUŠAN ĐJUKIĆ, SERBIAN NMO, 2017

42. Let p be a prime. Show that $\sqrt[3]{p}+\sqrt[3]{p^5}$ is irrational.

THAILAND NMO, 2017

43. Find all functions $f:\mathbb{R}\to\mathbb{R}$ which satisfy:

$$f(f(x)-y)\leq xf(x)+f(y) \ (1)$$

for all real numbers x and y.

THAILAND NMO, 2017

44. Find all functions $f: \mathbb{Q}^+ \to \mathbb{Q}^+$ such that:

$$f(xf(x) + f(y)) = (f(x))^2 + y \ (1)$$

for all positive rational x, y.

THAILAND NMO, 2017

45. Find all functions $f: \mathbb{N} \to \mathbb{N}$ such that for every positive integer m the following is true: If we denote by $d_1, d_2, ..., d_n$ all the divisors of number m, then:

$$f(d_1) \cdot f(d_2) \cdot ... \cdot f(d_n) = m.$$

PAVEL CALABEK, CZECH & SLOVAK NMO, 2017

46. Find all triplets of integers (a, b, c) such that each of the fractions:

$$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \text{ is an integer.}$$

JAROSLAV SVRCEK, CZECH & SLOVAK NMO, 2017

47. Let ABC be an acute triangle with altitude AD. The bisectors of angles BAD, CAD intersect side BC at E, F, respectively. The circumcircle of triangle AEF intersects sides AB, AC at G, H, respectively. Prove that lines EH, FG, and AD pass through a common point.

PATRIK BAK, CZECH & SLOVAK NMO, 2017

48. Let be $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \sum_{k=0}^{n} a_k x^k$, where $a_k \geq 0$, $\forall\ k = \overline{0,n}$. If $f(4) = 8$ and $f(9) = 18$, then find $\max(f(6))$ and the value for which this maximum is achieved.

D.M.BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

49. If $a, b, c > 0$, $a + b + c = 3$, $x \in \mathbb{R}$, then:

$$\left(\sqrt[3]{a\sin^2 x} + \sqrt[3]{b\cos^2 x}\right)\left(\sqrt[3]{b\sin^2 x} + \sqrt[3]{c\cos^2 c}\right)\left(\sqrt[3]{c\sin^2 x} + \sqrt[3]{a\cos^2 x}\right) \leq 4$$

DANIEL SITARU, RMM, ROMANIA

50. If $x, y, z > 0$, then:

$$\sum \frac{x^3}{y^3} + 2\sum \frac{y}{x} + 2\sum \frac{x}{y} + \sum \frac{y^3}{x^3} \geq \sum \frac{x^2}{y^2} + 12 + \sum \frac{y^2}{x^2}.$$

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51. If $x, y, z > 0$, $x + y + z = 3$, then:

$$\left(\sqrt[3]{x} + \sqrt[3]{y}\right)^3 + \left(\sqrt[3]{y} + \sqrt[3]{z}\right)^3 + \left(\sqrt[3]{z} + \sqrt[3]{x}\right)^3 \leq 24.$$

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52. If in ΔABC $S = 2$, then:

$$\frac{(2s-a)(2s-b)(2s-c)}{(2+a\sqrt{\sin A})(2+b\sqrt{\sin B})(2+c\sqrt{\sin C})} \geq \frac{1}{\sqrt{\sin A \sin B \sin C}}.$$

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53. In acute ΔABC, the following relationship holds:

$$a^2b^2c^2 \sin 2A \sin 2B \sin 2C \cos A \cos B \cos C \leq S^3.$$

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54. In ΔABC, the following relationship holds:

$$(am_a + bm_b + cm_c)(am_a^3 + bm_b^3 + cm_c^3) \leq (a+b+c)(am_a^4 + bm_b^4 + cm_c^4).$$

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55. If $a, b, c > 0$, then:

$$\frac{\sum \sqrt[3]{(a+3b)(2a+2b)(3a+b)}}{\sqrt{ab}+\sqrt{bc}+\sqrt{ca}} \geq 4.$$

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56. If $a, b, c > 0$, $a^2 + b^2 + c^2 = 3$, then:

$$\frac{a^3+1}{\sqrt{a^2-a+1}} + \frac{b^3+1}{\sqrt{b^2-b+1}} + \frac{c^3+1}{\sqrt{c^2-c+1}} \geq 6.$$

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57. In ΔABC, the following relationship holds:

$$\frac{(a+b)^4}{ab} \geq \frac{64s^2}{3}.$$

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58. If $a, b, c, d > 0$, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}=1$, then:

$$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}+\sqrt[3]{d} \leq \sqrt[3]{abcd}.$$

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59. If $x, y, z \geq 1$, then:

$$\sum \frac{1}{1+\sqrt{(x-1)(y-1)}} \leq \frac{3}{\sqrt[3]{xyz}}.$$

DANIEL SITARU, RMM, ROMANIA

60. In ΔABC, $\measuredangle B > \measuredangle C$. Let D be the point on side BC such that $\measuredangle DAC = \frac{B-C}{2}$. The circumcircle of ΔACD meets side AB again at E. The circumcircle of ΔABD meets side AC again at F. The internal angle bisector of $\measuredangle BDE$ meets side AB at P. The internal angle bisector of $\measuredangle CDE$ meets side AC at Q. Prove that PQ and AB are perpendicular.

HONG KONG, PREIMO 2017, MOCK EXAM

61. Let a, b, c, d be positive real numbers satisfying $abcd = 1$. Prove that:

$$(a^2b + b^2c + c^2d + d^2a)(ab^2 + bc^2 + cd^2 + da^2) \geq (a + c)(b + d)(ac + bd + 2).$$

When does equality hold?

HONG KONG, PREIMO 2017, MOCK EXAM

62. Prove the following inequality:

$[(x+y)(y+z)(z+x)]^4 \geq \frac{16^3}{27}(x+y+z)^3x^3y^3z^3$, where x, y, z are positive real numbers.

ANDREI BOGDAN UNGUREANU, RMM WINTER EDITION, 2016

63. Prove that if $a, b, c, d \in (0, \infty)$; $\sqrt{3}(ad - bc) = ac + bd \neq 0$, then:

$$d(a + b\sqrt{3}) - c(b - a\sqrt{3}) > 4\sqrt[4]{abcd}.$$

DANIEL SITARU, RMM WINTER EDITION, 2016

64. Prove that in an ABC acute-angled triangle the following relationship holds:

$$\cos\left(\frac{\pi}{4} - A\right) + \cos\left(\frac{\pi}{4} - B\right) + \cos\left(\frac{\pi}{4} - C\right) > \frac{2S}{R^2}.$$

DANIEL SITARU, RMM WINTER EDITION, 2017

65. Prove that in ΔABC:

$$\sum \frac{a^2(b^2+c^2-a^2)^3}{b^2c^2} \geq 64S^2(1 - \cos^2 A - \cos^2 B - \cos^2 C).$$

DANIEL SITARU, RMM WINTER EDITION, 2016

66. Let $x_1, x_2, ..., x_n$ be positive real numbers such that $\frac{1}{x_1} + \frac{2}{x_2} + ... + \frac{n}{x_n} = \frac{n(n+1)}{2}$.

Find the minimum possible value of $x_1 + x_2^2 + ... + x_n^n$.

NGUYEN VIET HUNG, RMM SPRING EDITION, 2017

67. Prove that for all $x \in \mathbb{R}$ we have $\cos(\sin x) > |\sin(\cos x)|$.

ABDALLAH EL FARISSI, RMM SPRING EDITION, 2017

68. Let $a, b \in \mathbb{R}$ such that $a + b > 0$, then:

$$\left(\frac{a+b}{2}\right)^n \le \frac{1}{n+1}\sum_{k=0}^{n} a^k b^{n-k} \le \frac{a^n + b^n}{2}.$$

ABDALLAH EL FARISSI, RMM SPRING EDITION, 2017

69. Call a function $f : \mathbb{N} \to \mathbb{N}$ lively if $f(a + b - 1) = \underbrace{f(f(...)f(b)...))}_{a \text{ times}}$ for all $a, b \in \mathbb{N}$.

Suppose that g is a lively function such that $g(A + 2018) = g(A) + 1$ holds for some $A \ge 2$.

a) Prove that $g(n + 2017^{2017}) = g(n)$ for all $n \ge A + 2$.

b) If $g(A + 2017^{2017}) \ne g(A)$, determine $g(n)$ for $n \le A - 1$.

MARKO RADOVANOVIĆ, SERBIAN TST, 2017

70. A $n \times n$ square is divided into unit squares. One needs to place a number of isosceles right triangles with hypotenuse 2, with vertices at grid points, in such a way that every side of every unit square belongs to exactly on triangle (i.e. lies inside it or on its boundary). Determine all numbers n for which this is possible.

DUŠAN ĐJUKIĆ, SERBIAN TST, 2017

71. Let $ABCD$ be a convex quadrilateral with $AC \perp BD$. Prove that there exist points P, Q, R, S on AB, BC, CD, DA, respectively, such that $PR \perp QS$ and the area of quadrilateral $PQRS$ is exactly half that of $ABCD$.

THAILAND TST, 2017

72. Let a and b be real numbers such that $a + b = 1$. Prove the following inequality:

$$\sqrt{1+5a^2} + 5\sqrt{2+b^2} \ge 9.$$

B. BATTSENGEL, MONGOLIAN NMO, 2017

73. Let $ABCD$ be an isosceles trapezoid with $AD = BC$ and $AB \parallel CD$. Let O be the intersection of the diagonals and let M be the midpoint of AD. Circumcircle of BCM intersects AD again at K. Prove that OK is parallel to AB.

B. BAT-OD, MONGOLIAN NMO, 2017

74. The altitudes AD and BE of acute triangle ABC intersect at H. Let F be the intersection of AB and a line that is parallel to the side BC and goes through the circumcenter of ABC. Let M be the midpoint of AH. Prove that $\sphericalangle CMF = 90°$.

G. BATZAYA, MONGOLIAN NMO, 2017

75. Let $ABCD$ be a cyclic quadrilateral with circumcenter w, and E be the intersection of the diagonals AC and BD. A line passing through E intersects lines AB, BC at P, Q, respectively. Let R ($R \neq D$) be the intersection point of w and circle that passes through D, E and tangents the line PQ at E. Prove that B, P, Q, R are cyclic.

B. KHOROLDAGVA, MONGOLIAN NMO, 2017

76. Find all functions $f: \mathbb{R} \to \mathbb{R}$ satisfying:

$$(a-b)f(a+b) + (b-c)f(b+c) + (c-a)f(c+a) = 0$$

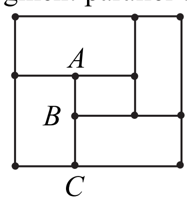
for all $a, b, c \in \mathbb{R}$.

MONGOLIAN NMO, 2017

77. Let I be the center of triangle ABC. Let D be a point on side BC, and E be a point on ray BC such that C lies between E and D and $\frac{BD}{DC} = \frac{BE}{EC}$. Let H be the feet of perpendicular from D to line IE. Prove that $\measuredangle AHE = \measuredangle IDE$.

B. BATTSENGEL, G. BATZAYA, MONGOLIAN NMO, 2017

78. A rectangle R is dissected into 2016 small rectangles with sides parallel to the sides of R. We call the vertices of those small rectangles nodes. A segment parallel to the sides of R is called basic if its two end points are nodes and there are no other nodes in the interior of it. Find the maximum and minimum of the number of basic segments among all possible dissections of R. For example, in the figure on the right, R is dissected into 5 small rectangles with a total of 16 segments. The segments AB and BC are basic, while the segment AC is not.



CHINA NMO, 2017

79. Let a, b be positive real numbers such that $a^2 + ab + b^2 = 9$. Find the maximal value of expression:

$$(a+b)^6 + (ab)^5 + 2(ab)^3 + (ab)^2 - 17.$$

GEORGE APOSTOLOPOULOS, RMM SUMMER EDITION, 2017

80. Let ABC be an equilateral triangle inscribed in the circle (O) whose radius is R. Prove that for an arbitrary point P lies on (O):

$$6\sqrt{2} < \frac{PA^3 + PB^3 + PC^3}{R^3} < 3\sqrt[4]{216}.$$

NGUYEN VIET HUNG, RMM SUMMER EDITION, 2017

81. If $a, b, c, n > 0$; $n(ab + bc + ca) + 2abc = n^3$, then:

$$\frac{1}{a+b+2n} + \frac{1}{b+c+2n} + \frac{1}{c+a+2n} \leq \frac{1}{n}.$$

MARIN CHIRCIU, RMM SUMMER EDITION, 2017

82. Prove that if $n \in \mathbb{N}^*$; $a > 1$, then:

$$(n + a - 1)(a - 1)^{n-1} \le a^n.$$

DANIEL SITARU, RMM SUMMER EDITION, 2017

83. Let ABC be an acute triangle. Prove that:

$$(a \cot A)^a (b \cot B)^b (c \cot C)^c \le (2r)^{a+b+c}.$$

where $a = BC$, $b = CA$, $c = AB$, and r is the in radius.

NGUYEN VIET HUNG, RMM AUTUMN EDITION, 2017

84. Let a, b, c be positive real numbers. Prove that:

$$\frac{a^3+b^3}{c^2+ab}+\frac{b^3+c^3}{a^2+bc}+\frac{c^3+a^3}{b^2+ca} \ge \frac{9abc}{ab+bc+ca}.$$

NGUYEN NGOC TU, RMM AUTUMN EDITION, 2017

85. Prove that if a, b, c are the length's sides in ΔABC, then:

$$\sin^2 a + \sin^2 b + \sin^2 c \ge 4 \sin s \cdot \sin(s-a) \cdot \sin(s-b) \cdot \sin(s-c).$$

DANIEL SITARU, RMM AUTUMN EDITION, 2017

86. If $u, v > 0$, with $2u - v > 0$ and α, β, γ are the measures of the angles of triangle ABC, then $\sum_{cyc} \frac{\sin\alpha}{u\sin\beta + v\sqrt{\sin\alpha + \sin\beta}} \ge \frac{3}{u+v}$.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU,
RMM AUTUMN EDITION, 2017

87. Let a, b, c be positive real numbers, take:

$$X = \frac{a}{b}+\frac{b}{a},\ Y = \frac{b}{c}+\frac{c}{b},\ Z = \frac{c}{a}+\frac{a}{c}.$$

Prove that: $X + Y + Z \ge 2\sqrt[4]{(X^2+Y^2+Z^2-3)(X+Y+Z+3)}$.

NGUYEN NGOC TU, RMM AUTUMN EDITION, 2017

88. The unit squares of an $N \times N$ board are coloured black and white so that squares that share a side have different colours, and so that at least one corner square is coloured black. In each step we choose a 2×2 square and change the colour of all four unit squares inside that square, so that white unit squares become black, black become grey, and grey become white.

Determine all positive integers $N > 1$ for which it is possible, using a finite number of steps, to achieve that all unit squares that were originally black become white, and all unit squares that were originally white become black.

CROATIAN NMO, 2017

89. If x, y, z and w are real numbers such that:

$$x^2 + y^2 + z^2 + w^2 + x + 3y + 5z + 7w = 4,$$

determine the largest possible value of $x + y + z + w$.

CROATIAN NMO, 2017

90. Determine the maximum value of the expression:

$$\sin x \sin y \sin z + \cos x \cos y \cos z.$$

CROATIAN NMO, 2017

91. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that:

$$f(x + f(y)) = f(f(y)) + 2xf(y) + x^2$$

holds for all real numbers x and y.

TONCI KOKAN, CROATIAN NMO, 2017

92. Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that:

a) $\dfrac{1}{a+bc} + \dfrac{1}{b+ca} + \dfrac{1}{c+ab} \le \dfrac{1}{4abc}$;

b) $\dfrac{\sqrt{a}}{a+\sqrt{bc}} + \dfrac{\sqrt{b}}{b+\sqrt{ca}} + \dfrac{\sqrt{c}}{c+\sqrt{ab}} \le \dfrac{1}{2\sqrt{abc}}$.

NGUYEN VIET HUNG, RMM WINTER EDITION, 2017

93. Let a, b, c positive numbers such that $a^4 + b^4 + c^4 = 3$. Prove that:

$$\left(\frac{a^3}{b^5} + \frac{b^3}{c^5} + \frac{c^3}{a^5}\right)\left(\frac{b^3}{a^5} + \frac{c^3}{b^5} + \frac{a^3}{c^5}\right) \ge 9.$$

NGUYEN NGOC TU, HA GIANG, VIETNAM

94. Let $a, b, c > 0$ such that $(a + b)(b + c)(c + a) = 8$. Prove that:

$$\frac{a}{a+1} + \sqrt{\frac{2b}{b+1}} + 2\sqrt[4]{\frac{2c}{c+1}} \le \frac{7}{2}.$$

NGUYEN NGOC TU, HA GIANG, VIETNAM

95. Let $m > 0$ and F be the area of the triangle ABC. Then:

$$\frac{a^{m+2}}{b^m + c^m} + \frac{b^{m+2}}{c^m + a^m} + \frac{c^{m+2}}{a^m + b^m} \ge 2\sqrt{3}\,\mathrm{F}.$$

D.M. BĂTINEȚU-GIURGIU, ROMANIA, MARTIN
LUKAREVSKI, RMM WINTER EDITION, 2017

96. Solve for real numbers:

$$n^{n(x_1^2 - x_2)} + n^{n(x_2^2 - x_3)} + \ldots + n^{n(x_{n-1}^2 - x_n)} + n^{n(x_n^2 - x_1)} = \frac{n}{\sqrt[4]{n^n}}.$$

DANIEL SITARU, RMM WINTER EDITION, 2017

97. Inside a circle of radius 1 (or on the circumference), one marks n points in such a way that the minimal distance between two marked points is as large as possible. Let d_n be this distance between the two closest points. Is it true that $d_{n+1} < d_n$ for every natural number $n \geq 2$?

ESTONIAN NMO, 2017

98. Find all functions $f : \mathbb{R} \to \mathbb{R}$ that for all numbers x and y satisfy $f(x + y)f(xy) = f(x^2 - y^2 + 1)$.

ESTONIAN NMO, 2017

99. Solve the system of equations $3x + 7y + 14z = 252$, $xyz - u^2 = 2016$ for non-negative real numbers.

ESTONIAN NMO, 2017

100. Find all positive integers n such that a square can be cut into n square pieces.

ESTONIAN NMO, 2017

101. Real numbers x, y and z satisfy $x + y + z = 4$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{3}$. Find the largest and the smallest possible value of the expression $x^2 + y^3 + z^3 + xyz$.

ESTONIAN NMO, 2017

102. Solve for real numbers:

$$\begin{cases} \tan x \tan y \tan z = 6 \\ \tan x \tan y + \tan x \tan z + \tan y \tan z = 11. \\ x + y + z = \pi \end{cases}$$

DANIEL SITARU, RMM, ROMANIA

103. Find $x, y, z \in (0, \infty)$ such that:

$$\begin{cases} x + y + z = xyz \\ \dfrac{x}{y^3 z^2} + \dfrac{y}{z^3 x^2} + \dfrac{z}{x^3 y^2} = \dfrac{1}{3}. \end{cases}$$

DANIEL SITARU, RMM, ROMANIA

104. Solve for real numbers:

$$\begin{cases} 1 + 2\sqrt{y} = 2\sqrt{x+1} \\ \dfrac{2\sqrt{y}}{12y+1} + \dfrac{\sqrt{x+1}}{x+4} + \dfrac{2\sqrt{y(x+1)}}{3x+3y+3} = \dfrac{3}{4}. \end{cases}$$

NGO MINH NGOC BAO, RMM, VIETNAM

105. Solve the system of equations:

$$\begin{cases} \sqrt{x}+\sqrt{y}+\sqrt{z}+1=4\sqrt{xyz} \\ xy+yz+zx+3=2\cdot(\sqrt[4]{x}+\sqrt[4]{y}+\sqrt[4]{z}) \end{cases} \quad (1).$$

HOANG LE NHAT TUNG, RMM, VIETNAM

106. Solve for integers:

$$\begin{cases} x(y+z)=y^2+z^2-6 \\ y(z+x)=z^2+x^2-6 \\ z(x+y)=x^2+y^2-6 \end{cases}.$$

MARIN CHIRCIU, RMM, ROMANIA

107. Solve for real numbers:

$$\begin{cases} \cos 2x+\cot 3y=\tan 5z \\ \cot 3y+\cot 5z=\tan 2x \\ \cot 5z+\cot 2x=\tan 3y \end{cases}.$$

DANIEL SITARU, RMM, ROMANIA

108. Suppose a sequence of integers $x_1, x_2, \dots$ satisfies the following conditions:

- $x_1 = 1, x_2 = x_3 = \dots = x_{13} = 0$.
- $x_{n+13} = x_{n+5} + 2x_n$ $(n = 1, 2, \dots)$.

Find the value of x_{144}.

JAPAN NMO, 2017

109. Solve the question in $\mathbb{R}$:

$$\sqrt{x^3-2x^2+2x}+3\cdot\sqrt[3]{x^2-x+1}+2\cdot\sqrt[4]{4x-3x^4}=\frac{x^4-3x^3}{2}+7. \quad (1)$$

HOANG LE NHAT TUNG, RMM, VIETNAM

110. Solve for real numbers:

$$\arcsin[x]\cdot\arccos[x]=\frac{\pi x}{2}-x^2.$$

ROVSEN PIRGULIEV, RMM, AZERBAIJAN

111. Solve for real numbers:

$$[\tan x]\cdot(\cot x-[\cot x])=(\tan x-[\tan x])\cdot[\cot x]$$

[*] – great integer function.

ROVSEN PIRGULYEV, RMM, AZERBAIJAN

112. Prove that:

$$\sin^2\frac{7\pi}{18}\cdot\sin\frac{5\pi}{18}-\sin^2\frac{\pi}{18}\cdot\sin\frac{7\pi}{18}+\sin^2\frac{5\pi}{18}\cdot\sin\frac{\pi}{18}=\frac{3}{4}.$$

GEORGE APOSTOLOPOULOS, RMM, GREECE

113. Find $A, B, C \in (0, \pi)$, $A + B + C = \pi$ such that:

$$\begin{cases}\cos A|\cos B|+\cos B|\cos A|=1+\cos 2C\\ \cos B|\cos C|+\cos C|\cos B|=1+\cos 2A\\ \cos C|\cos A|+\cos A|\cos C|=1+\cos 2B\end{cases}.$$

DANIEL SITARU, RMM, ROMANIA

114. Solve for real numbers:

$$\begin{cases}27^x+2=3^{y+1}\\ 27^y+2=3^{z+1}\\ 27^z+2=3^{x+1}\end{cases}.$$

SEYRAN IBRAHIMOV, RMM, AZERBAIJAN

115. For each positive real number x, we define $\{x\}$ to be the greater of x and $\frac{1}{x}$, with $\{1\} = 1$. Find, with proof, all positive real numbers y such that $5y\{8y\}\{25y\} = 1$.

DAN GRILLER, BRITISH NMO, 2017

116. Naomi and Tom play a game, with Naomi going first. They take it in turns to pick an integer from 1 to 100, each time selecting an integer which no one has chosen before. A player loses the game if, after their turn, the sum of all the integers chosen since the start of the game (by both of them) cannot be written as the difference of two square numbers. Determine if one of the players has a winning strategy, and if so, which.

TOM BOWLER, BRITISH NMO, 2017

117. If $m \in [0, \infty)$; $a, b, x, y, z \in (0, \infty)$, then:

$$\frac{x^{2m+2}}{(ay+bz)^{2m+2}\sec^{2m}\frac{\pi}{18}}+\frac{y^{2m+2}}{(az+bx)^{2m+2}\csc^{2m}\frac{\pi}{9}}+\frac{z^{2m+2}}{(ax+by)^{2m+2}\csc^{2m}\frac{\pi}{9}}\ge\frac{3}{4^m(a+b)^{2m+2}}.$$

D.M. BĂTINEȚU-GIURGIU, DANIEL SITARU,
LA GACETA DE LA RSME, 2017

118. Given a positive integer n, define $f(0, j) = f(i, 0) = 0$, $f(1, 1) = n$, and $f(i, j) = \left[\frac{f(i-1,j)}{2}\right]+\left[\frac{f(i,j-1)}{2}\right]$ for all positive integers i and j, $(i, j) \neq (1, 1)$. How many ordered pairs of positive integers (i, j) are there for which $f(i, j)$ is an odd number?

DUŠAN ĐJUKIĆ, SERBIAN NMO, 2016

119. Solve for real numbers:

$$\begin{cases} \sin[x]+\cos(x-[x])=\dfrac{\sqrt{3}}{2} \\ \sin(x-[x])+\cos[x]=\dfrac{3}{2} \end{cases}, \ [*] - \text{great integer function.}$$

ROVSEN PIRGULIYEV, RMM, AZERBAIJAN

120. Solve for $x > 0$ the equation:

$$e^x+\pi^x+\frac{1}{e^x}+\frac{1}{\pi^x}=\frac{1}{\cot^{-1}(e^x)}+\frac{1}{\cot^{-1}(\pi^x)}.$$

ROVSEN PIRGULIYEV, RMM, AZERBAIJAN

121. If $a, r \in (0, \infty)$, then:

$$\sum_{k=1}^{n}\frac{k}{\left(\sum_{i=1}^{k}\left(\frac{1}{a+(i-1)r}\right)\right)}<(2a+(n-1)r)n, \ n\in\mathbb{N}^*.$$

DANIEL SITARU, RMM, ROMANIA

122. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that:

$$f(xf(y)-f(x))=2f(x)+xy \ (1).$$

VIETNAMESSE NMO, 2017

123. Prove that in any right-angled triangle the following inequality holds:

$$\frac{m_a m_b m_c}{abc}\geq\frac{5}{8}.$$

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

124. Find all the functions $f:\mathbb{N}^*\to[1,\infty)$, which satisfy the conditions:

(i) $f(2) = 2$; (ii) $f(n) \leq f(n + 1)$; (iii) $f(nm) = f(n)f(m)$.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

125. If $x_1, x_2, ..., x_n$ are real positive numbers, prove that:

$$\frac{1}{2^n\sqrt{x_1x_2\cdot\ldots\cdot x_n}}+\sum_{k=1}^{n}\frac{x_k}{(x_1+1)(x_2+1)\cdot\ldots\cdot(x_k+1)}\geq 1.$$

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

126. Let be the sequences $(a_n)_{n\geq 2}$, $(b_n)_{n\geq 2}$, $(c_n)_{n\geq 2}$ defined:

$$a_2=3,\ b_2=2,\ a_{n+1}=a_n^2+2b_n^2,\ b_{n+1}=2a_nb_n,\ c_n=\sqrt{\frac{b_n^2}{a_n^2-2b_n^2}},\forall n\geq 2.$$

Prove that the sequence $(c_n)_{n\geq 2}$ has all the terms natural numbers.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

127. Solve in $\mathbb{Z} \times \mathbb{Z}$ the equation: $x^2 - 2x + 2xy - 2y + y^2 = 3$.

MARIN CHIRCIU, ROMANIA

128. Solve for real numbers: $x\sqrt{4+x} + 2\sqrt{21-x} = 5\sqrt{x^2+4}$.

MARIN CHIRCIU, ROMANIA

129. Let be $a > 0$. Solve for real numbers:

$$x\sqrt{a+x} + \sqrt{a^3+a^2+a-ax} = (a+1)\sqrt{x^2+a}.$$

MARIN CHIRCIU, ROMANIA

130. In $ABCD$ cyclic quadrilater, $AB = a$, $BC = b$, $CD = c$, $DA = d$, S – area $[ABCD]$.

$$\sin A + \sin B + \sin C + \sin D \leq \frac{4S}{\sqrt{abcd}}.$$

DANIEL SITARU, RMM, ROMANIA

131. In $ABCD$ cyclic quadrilater, $AB = a$, $BC = b$, $CD = c$, $DA = d$, s – semiperimeter:

$$\sin A \sin B \leq \left(1-\frac{s}{a}\right)\left(1-\frac{s}{b}\right)\left(1-\frac{s}{c}\right)\left(1-\frac{s}{d}\right).$$

DANIEL SITARU, RMM, ROMANIA

132. In $ABCD$ convexes quadrilateral: $AB = a$, $BC = b$, $CD = c$, $DA = d$. Prove that:

$$\sum\sqrt{a^2+b^2+c^2} > 2\sqrt{3\cdot AC\cdot BD}.$$

DANIEL SITARU, RMM, ROMANIA

133. If $x, y, z \in [0, \infty)$, then:

$$\sqrt{x^2 - xy\sqrt{3} + y^3} + \sqrt{y^2 - yz\sqrt{2} + z^2} \geq \sqrt{x^2 - xz + z^2}.$$

DANIEL SITARU, RMM, ROMANIA

134. If $x, y, z \in (0, \infty)$, $xyz = 1$, then:

$$x(x - 3(y+z))^2 + (3x - (y+z))^2(y+z) \geq 27.$$

DANIEL SITARU, RMM, ROMANIA

135. If $a, b, c > 0$, then:

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{c+a}{c+b}+\frac{a+b}{a+c}+\frac{b+c}{b+a}.$$

GEORGE APOSTOLOPOULOS, RMM, GREECE

136. If $a, b, c > 0$, $a + b + c = 3$, then:

$$\frac{a^4}{b^4\sqrt{2c(a^3+1)}}+\frac{b^4}{c^4\sqrt{2a(b^3+1)}}+\frac{c^4}{a^4\sqrt{2b(c^3+1)}}\geq\frac{a^2+b^2+c^2}{2}.$$

HOANG LE NHAT TUNG, RMM, VIETNAM

137. If $a, b, c > a^6 + b^6 + c^6 = 9$, then:

$$2\left(\frac{a+b}{(a^3\sqrt{b}+b^2\sqrt{a})^2}+\frac{b+c}{(b^3\sqrt{c}+c^3\sqrt{b})^2}+\frac{c+a}{(c^3\sqrt{a}+a^3\sqrt{c})^2}\right)\geq 1.$$

DANIEL SITARU, RMM, ROMANIA

138. Let be $0 < a < b < c$. Prove that:

$$\sum(e^{a-b}+e^{b-a})>2a-2c+3+\sum\left(\frac{b}{a}\right)^{\sqrt{ab}}.$$

DANIEL SITARU, SSMA MAGAZINE, 2017

139. If $a, b, c > 0$, $a \neq b \neq c \neq a$, then:

$$\frac{1}{(a-b)^2}+\frac{1}{(b-c)^2}+\frac{1}{(c-a)^2}+\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}>\frac{81}{4(a^2+b^2+c^2)}.$$

DANIEL SITARU, RMM, ROMANIA

140. If $a, b, c > 0$, $a + b + c = 1$, then:

$$a^3+b^3+c^3+6abc\geq a^{a^2+2bc}\cdot b^{b^2+2ac}\cdot c^{c^2+2ab}.$$

DANIEL SITARU, RMM, ROMANIA

141. Prove that if $x, y, z \in [1, 2]$, then in ΔABC:

$$\sqrt[3]{abcxyz}\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)^2\leq\frac{16s^3}{3}.$$

DANIEL SITARU, RMM, ROMANIA

142. If $x, y, z \geq 0$ such that $x^3 + y^3 + z^3 = 3$ and $n \geq \frac{4}{3}$, prove that:

$$\frac{x+n}{nx^2+1}+\frac{y+n}{ny^2+1}+\frac{z+n}{nz^2+1}\geq 3.$$

MARIN CHIRCIU, ROMANIA

143. Solve for real numbers:

$$\frac{\pi}{2}+\arcsin\frac{2x^2-a^2}{a^2}=2\arcsin\frac{x}{a}\text{, where } a>1,\ a \text{ is given.}$$

MARIN CHIRCIU, ROMANIA

144. If $a, b, c > 0$ such that $a + b + c = 1$ and $n \in \mathbb{N}^*$, prove that:

$$\frac{a^{n+1}}{1-a^n}+\frac{b^{n+1}}{1-b^n}+\frac{c^{n+1}}{1-c^n}\geq\frac{1}{3^n-1}.$$

MARIN CHIRCIU, OCTAVIAN STROE, ROMANIA

145. Let O be circumcenter of ABC triangle. Prove that:

$$\overrightarrow{AO}\cdot\overrightarrow{AB}+\overrightarrow{BO}\cdot\overrightarrow{BC}+\overrightarrow{CO}\cdot\overrightarrow{CA}=\frac{a^2+b^2+c^2}{2}.$$

MARIN CHIRCIU, ROMANIA

146. Prove that in any triangle ABC and for all positive real numbers x, y, z the following inequality holds:

$$\left(\frac{x}{r_a}+\frac{y}{r_b}+\frac{z}{r_c}\right)\left(\frac{x}{r_b}+\frac{y}{r_c}+\frac{z}{r_a}\right)\left(\frac{x}{r_c}+\frac{y}{r_a}+\frac{z}{r_b}\right)\geq\frac{xyz}{r^3}.$$

HUNG NGUYEN VIET, RMM, VIETNAM

147. Prove that in any triangle ABC the following relationship holds:

$$\frac{\cot\frac{B}{2}\cot\frac{C}{2}}{\cot\frac{A}{2}}+\frac{\cot\frac{C}{2}\cot\frac{A}{2}}{\cot\frac{B}{2}}+\frac{\cot\frac{A}{2}\cot\frac{B}{2}}{\cot\frac{C}{2}}\geq 2\left(\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}\right).$$

HUNG NGUYEN VIET, RMM, VIETNAM

148. Let ABC be a triangle with $a = BC$, $b = CA$, and $c = AB$. Let $A'B'C'$ be another triangle with $B'C' = \sqrt{a}$, $C'A' = \sqrt{b}$, $A'B' = \sqrt{c}$. Prove that:

$$\sin\left(\frac{1}{2}A\right)\sin\left(\frac{1}{2}B\right)\sin\left(\frac{1}{2}C\right)=\cos A'\cos B'\cos C'.$$

MEHMET ȘAHIN, RMM, TURKEY

149. In acute-angled ΔABC, the following relationship holds:

$$\sum(\sin 2A+\sin B)\left(\frac{1}{\sin 2A}+\frac{1}{\sin 2B}\right)\leq\sum(\tan A+\tan B)(\cot A+\cot B).$$

DANIEL SITARU, RMM, ROMANIA

150. Let be $a, b, c \in (1, \infty)$. Solve for real numbers:

$$\left(a^x + b^{\frac{1}{x}} + c^{x+\frac{1}{x}}\right)\left(a^{\frac{1}{x}} + b^x + c^{x+\frac{1}{x}}\right) = (a+b+c^2)^2 \ .$$

MARIN CHIRCIU, ROMANIA

151. Let be $n \in \mathbb{N}^*$. Find $x \geq 0$ with the property:

$$x^{n+1} + n = (n+1)\sqrt[n+1]{(n+1)x - n}\ .$$

MARIN CHIRCIU, ROMANIA

152. Prove that the sequence $(a_n)_{n\,\geq\,1}$ defined by $a_{n+1} = \dfrac{a_{n-1}\cdot a_n}{2a_{n-1} - a_n}$ has a term that is the reverse of a perfect square, then the sequence $(b_n)_{n\,\geq\,1}$ defined by $b_n = \dfrac{1}{a_n}$ contains an infinity of perfect squares.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

153. If $a_n = 1 + 2011n$ ($n \in \mathbb{N}$), then prove that:

a) $(a_n)_{n\,\geq\,0}$ contains an infinity of perfect squares;
b) $(a_n)_{n\,\geq\,0}$ contains an infinity of perfect squares;
c) $(a_n)_{n\,\geq\,0}$ contains an infinity of natural numbers having the form $a^2 + a + 1$ ($a \in \mathbb{N}$).

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

154. If $f : \mathbb{N} \to \mathbb{N}$, such that for any $1 < x \leq y \leq z$ we have $f(x) + f(y) = z$, $f(y) + f(z) = x$, $f(z) + f(x) = y$ and for any $z > 2$ we have $f(z) > 1 + \dfrac{z}{2}$, then find all the triplets (x, y, z).

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

155. Prove that if $a + b \neq 0$ such that $a \cos ax + b \cos bx = c \cos cx + d \cos dx$, $\forall\, x \in \mathbb{R}$, then $a = c$ or $a = d$.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

156. Let be $n \in \mathbb{N}^*$ and $k \in \mathbb{N}$, $k \geq 2$. Solve the system:

$$\begin{cases} \sqrt[2n]{x\sqrt{x}} - \sqrt[2n]{y\sqrt{y}} = k^3 - 1 \\ \sqrt[7n]{x\sqrt{x\sqrt{x}}} + \sqrt[7n]{y\sqrt{y\sqrt{y}}} = k + 1 \end{cases}.$$

MARIN CHIRCIU, ROMANIA

157. The real positive numbers x, y, z, n have the property that $xy, yz, zx \le n$. Prove that if:

$$\frac{(n+x^2)(n+y^2)}{2n+x^2+y^2}+\frac{(n+y^2)(n+z^2)}{2n+y^2+z^2}+\frac{(n+z^2)(n+x^2)}{2n+z^2+x^2}\ge\frac{3n+xy+yz+zx}{2}.$$

MARIN CHIRCIU, ROMANIA

158. If $f:\mathbb{R}\to\mathbb{R}$, such that $f(\operatorname{ctg} x)=\sin 2x+\cos 2x$, $\forall\, x\in(0,\pi)$, then find $f(x)$.

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, ROMANIA

159. Prove that for any triangle ABC and all positive real numbers x, y, z the following inequality holds:

$$\frac{x}{y+z}\cdot\frac{\sin A}{\sin B\sin C}+\frac{y}{z+x}\cdot\frac{\sin B}{\sin C\sin A}+\frac{z}{x+y}\cdot\frac{\sin C}{\sin A\sin B}\ge\sqrt{3}.$$

NGUYEN VIET HUNG, RMM, VIETNAM

160. In ΔABC, the following relationship holds:

$$\sin^2 A\cos^4 A+\sin^2 B\cos^4 B+\sin^2 C\cos^4 C<\frac{4}{9}.$$

DANIEL SITARU, RMM, ROMANIA

161. In ΔABC, the following relationship holds:

$$3\left(\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}\right)\ge\sum\frac{b^2+bc+c^2}{bcm_a}.$$

DANIEL SITARU, RMM, ROMANIA

162. Prove that in any ΔABC the following relationship holds:

$$\frac{m_am_bm_c}{r_ar_br_c}+\frac{w_aw_bw_c}{h_ah_bh_c}\le\frac{R}{r}.$$

DANIEL SITARU, RMM, ROMANIA

163. In ΔABC, the following relationship holds:

$$2(AN^2+BN^2+CN^2)+42Rr\le 4s^2+3r\cdot ON$$

N – Nagel's point, O – circumcentre, s – semiperimeter, r – inradius, R – circumradius.

DANIEL SITARU, RMM, ROMANIA

164. In acute triangle ABC, the following relationship holds:

$$\frac{\tan^4 A}{\tan^3 B}+\frac{\tan^4 B}{\tan^3 C}+\frac{\tan^4 C}{\tan^3 C}\ge\tan A\tan B\tan C.$$

DANIEL SITARU, RMM, ROMANIA

165. In ΔABC, the following relationship holds:

$$\frac{1}{r^3}\sum a^3\cos B\cos C \geq 16\left(\sum \sin A\right)\left(\sum \cos^2 A\right).$$

DANIEL SITARU, RMM, ROMANIA

166. In ΔABC, the following relationship holds:

$$(m_a + r_a)(m_b + r_b)(m_c + r_c) \geq 8w_a w_b w_c.$$

DANIEL SITARU, RMM, ROMANIA

167. In ΔABC, the following relationship holds:

$$\frac{r_a^2}{a}+\frac{r_b^2}{b}+\frac{r_c^2}{c}\geq \frac{81r^2}{2p}.$$

DANIEL SITARU, RMM, ROMANIA

168. In ΔABC, I – incentre, the following relationship holds:

$$s^2\sum AI^2 > m_a m_b w_a w_b + m_b m_c w_b w_c + m_c m_a w_c w_a.$$

DANIEL SITARU, RMM, ROMANIA

169. In ΔABC, the following relationship holds:

$$\left(m_a^7 + m_b^7 + m_c^7\right)\left(\frac{1}{m_a^3}+\frac{1}{m_b^3}+\frac{1}{m_c^3}\right)\geq s^4,\ s - \text{semiperimeter}.$$

DANIEL SITARU, RMM, ROMANIA

170. In ΔABC, the following relationship holds:

$$16\left(\sum\frac{r_a^6}{a^3}\right)\left(\sum\frac{a^3}{r_a^2}\right)\geq 9(a^2+b^2+c^2)^2.$$

DANIEL SITARU, RMM, ROMANIA

171. Prove that in any triangle the following relationship holds:

$$\sqrt[3]{(r_a+r_b)(r_b+r_c)(r_c+r_a)}\geq \frac{2p}{\sqrt{3}}.$$

ADIL ABDULLAYEV, RMM, AZERBAIJAN

172. Prove that in any triangle the following relationship holds:

$$\sum_{cyc}\sqrt{r_a^2+1}\geq \sqrt{6(4R+r)}.$$

ADIL ABDULLAYEV, RMM, AZERBAIJAN

173. In ΔABC, the following relationship holds:

$$3\left(\sum \frac{a^3}{w_a}\right)\cdot\left(\sum \frac{w_a}{a}\right) \ge 4\left(\sum w_a\right)^2.$$

DANIEL SITARU, RMM, ROMANIA

174. Let $S = \{1, 2, ..., n\}$, $n \ge 2$, and let $f : S \to S$ be a bijective function distinct from the identity. Let $u = \sum_{k=1}^{n} |f(k) - k|$ and let it be the number of ordered pairs (a, b) of elements of S such that $a > f(a) < f(b)$. Show that $v < u \le 2v$, and that $u = 2v$ if and only if there do not exist positive integers $a > b > c$ such that $f(a) < f(b) < f(c)$.

JOSÉ LUIS DÍAZ-BARRERO, BARCELONA TECH, MATH CONTEST, 2014

175. For real numbers $x_1 > x_2 > ... > x_n \ge 0$, $n \ge 2$, that satisfy a condition $x_1 + x_2 + + ... + x_n = n$, prove an inequation:

$$2(x_1x_2 + x_1x_3 + ... + x_1x_n + x_2x_3 + x_2x_4 + ... + x_2x_n + ... + x_{n-1}x_n) \ge n \cdot (x_2 + x_3 + ... + x_n).$$

VITALY SENIN, UKRAINIAN NMO, 2017

176. Find all the functions $f : \mathbb{R} \to \mathbb{R}$ such that $\forall\, x, y \in \mathbb{R}$ the equation is true:

$$f(x + f(f(y))) = y + f(f(x)).$$

ANDRIY ANIUKUSHIN, UKRAINIAN NMO, 2017

177. Let $\overline{abc}$ be a prime number. Prove that equation $ax^2 + bx + c = 0$ does not have rational roots.

JOSÉ LUIS DÍAZ-BARRERO, BARCELONA TECH, MATH CONTEST, 2015

178. Let a, b, c be three positive numbers such that $ab + bc + ca = 3abc$. Prove that:

$$\sqrt{\frac{a+b}{c(a^2+b^2)}} + \sqrt{\frac{b+c}{a(b^2+c^2)}} + \sqrt{\frac{c+a}{b(c^2+a^2)}} \le 3.$$

JOSÉ LUIS DÍAZ-BARRERO, BARCELONA TECH, MATH CONTEST, 2017

179. If $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$ such that $\alpha + \beta + \gamma = \frac{\pi}{2}$ and $a \ge 0$, prove that:

$$\sqrt{a + \operatorname{tg}\alpha \operatorname{tg}\beta} + \sqrt{a + \operatorname{tg}\beta \operatorname{tg}\gamma} + \sqrt{a + \operatorname{tg}\gamma \operatorname{tg}\alpha} \le \sqrt{9a + 3}.$$

MARIN CHIRCIU, ROMANIA

180. If $a, b, c \ge a^2 + b^2 + c^2 = 2$, then:

$$\sqrt{ab}\csc\frac{\pi}{7} + \sqrt{bc}\csc\frac{2\pi}{7} + \sqrt{ca}\csc\frac{3\pi}{7} \le 4.$$

DANIEL SITARU, RMM, ROMANIA

181. If $n \in \mathbb{N}^*$, $n \geq 2$, $a, b, c > 1$, $a + b + c = 3^{n+1}$, then:

$$\sum\left(\sqrt[n]{a+\sqrt[n]{a}}+\sqrt[n]{a-\sqrt[n]{a}}\right)<18.$$

DANIEL SITARU, RMM, ROMANIA

182. If $m \in \mathbb{N}$, $m \geq 2$, then:

$$m+\tan^2\frac{\pi}{36}+\tan^2\frac{11\pi}{36}+\tan^2\frac{13\pi}{36}+\tan^2\frac{21\pi}{36}>2+\left(\frac{3}{16}\right)^{\frac{m-2}{m}}.$$

DANIEL SITARU, RMM, ROMANIA

183. If $a, b \in \left(0, \frac{\pi}{2}\right)$, then:

$$\frac{\cos a}{1+\cos^4 a}+\frac{\sin a\cos b}{1+\sin^4 a\cos^4 b}+\frac{\sin a\sin b}{1+\sin^4 a\sin^4 b}\leq\frac{9\sqrt{3}}{10}.$$

DANIEL SITARU, RMM, ROMANIA

184. If $a, b, c, x \in \mathbb{R}$, then:

$a^2 + b^2 + c^2 + (\sin x + \cos x + \sin x \cos x)(ab + bc + ca) \geq 0.$

DANIEL SITARU, RMM, ROMANIA

185. Let $OABC$ be a tetrahedron with $\measuredangle AOB = \measuredangle BOC = \measuredangle COA = 90°$ and let P be any point inside the triangle ABC. Denote, respectively, by d_a, d_b, d_c the distances from P to faces (OBC), (OCA), (OAB). Prove that:

a) $d_a^2 + d_b^2 + d_c^2 = OP^2$;

b) $d_a d_b d_c \leq \dfrac{OA \cdot OB \cdot OC}{27}$;

c) $OA \cdot d_a^3 + OB \cdot d_b^3 + OC \cdot d_c^3 \geq OP^4$.

NGUYEN VIET HUNG, RMM, VIETNAM

186. If $x, y > 0$; $z \in \mathbb{R}$, then:

$$\frac{(x+y)^2}{(x\sin^2 z+y\cos^2 z)(x\cos^2 z+y\sin^2 z)}+\frac{x}{y}+\frac{y}{x}\geq 6.$$

DANIEL SITARU, RMM, ROMANIA

187. In ΔABC:

$$\frac{\sin^2 A}{\sin^{-1}\frac{4}{5}}+\frac{\sin^2 B}{\sin^{-1}\frac{5}{13}}+\frac{\sin^2 C}{\sin^{-1}\frac{16}{65}}\geq\frac{2s^2}{\pi R^2}$$

$$\frac{\sin^2 A}{\tan^{-1}\frac{1}{2}}+\frac{\sin^2 B}{\tan^{-1}\frac{1}{5}}+\frac{\sin^2 C}{\tan^{-1}\frac{1}{8}}\ge\frac{4s^2}{\pi R^2}.$$

DANIEL SITARU, RMM, ROMANIA

188. If $a, b, c, d > 0$, then:

$$(a+b)^2(a+c)^2(a+d)^2(b+c)^2(b+d)^2(c+d)^2 \ge$$
$$\ge\left(a+\sqrt[3]{bcd}\right)^3\left(b+\sqrt[3]{cda}\right)^3\left(c+\sqrt[3]{dab}\right)^3\left(d+\sqrt[3]{abc}\right)^3.$$

MIHÁLY BENCZE, RMM, ROMANIA

189. If $a, b, c, x, y, z > 0$, $a + b + c \ge 3$, then:

$$(ax+by+cz)(ay+bz+cx)(az+bx+cy)\ge\frac{729}{\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)^3}.$$

DANIEL SITARU, RMM, ROMANIA

190. If $a, b, c > 0$, $a + b + c = 3$, $x \in \mathbb{R}$, then:

$$\left(\sqrt[3]{a\sin^2 x}+\sqrt[3]{b\cos^2 x}\right)\left(\sqrt[3]{b\sin^2 x}+\sqrt[3]{c\cos^2 c}\right)\left(\sqrt[3]{c\sin^2 x}+\sqrt[3]{a\cos^2 x}\right)\le 4.$$

DANIEL SITARU, RMM, ROMANIA

191. If $x, y \ge 0$, $n \ge 1$, $n \in \mathbb{Q}$, $AM=\frac{x+y}{2}$, $GM=\sqrt{xy}$, then:

$$\left(\frac{x^n+y^n}{\sqrt{2}}\right)^2\ge AM^{2n}+GM^{2n}.$$

UCHE ELIEZER OKEKE, RMM, NIGERIA

192. If $x, y, z \in \left(0,\frac{\pi}{2}\right)$, then:

$$\prod\ln(1+\tan^2 x)\cdot\prod\ln(1+\cot^2 y)\le\prod\ln^2\left(\frac{2}{\sin 2z}\right).$$

DANIEL SITARU, RMM, ROMANIA

193. Prove that $2^{\cos x}+2^{\sin x}\ge 2^{\frac{\sqrt{2}-1}{\sqrt{2}}}$, $\forall\, x \in \mathbb{R}$.

IBRAHIM ABDULAZEEZ, RMM, NIGERIA

194. Prove that in any triangle the inequality is true:

$$(na-p)(nb-p)(nc-p) \le \left(\frac{2n-3}{3}\cdot p\right)^3, \text{ where } n \ge 2.$$

MARIN CHIRCIU, ROMANIA

195. Prove that in any triangle the following inequality holds:

$$n\prod \sin^2 A + \prod \cos^2 A \le \frac{27n+1}{56}\left(1-\prod \cos A\right), \text{ where } 1 \le n \le 3.$$

MARIN CHIRCIU, ROMANIA

196. Prove that in any triangle the following inequality holds:

$$\frac{n\cdot R + k\cdot r}{\sqrt{3}} \ge \frac{p^2 + (6n+3k-27)r^2}{p}, \text{ where } n \ge 4 \text{ and } k \ge 1.$$

MARIN CHIRCIU, ROMANIA

197. Prove that in any triangle the following inequality is true:

$$\sqrt{\frac{n(a^2+b^2+c^2)+k(ab+bc+ca)}{n+k}} \ge \frac{2}{3}(m_a+m_b+m_c),$$

where n and k are positive numbers with $2n \ge 7k$.

MARIN CHIRCIU, ROMANIA

198. Prove that in any triangle the following inequality is true:

$$\frac{a}{\sqrt{p-na}} + \frac{b}{\sqrt{p-nb}} + \frac{c}{\sqrt{p-nc}} \ge 2\sqrt{\frac{3p}{3-2n}}, \text{ where } 0 \le n \le 1.$$

MARIN CHIRCIU, ROMANIA

199. Prove that in any triangle the following inequality holds:

$$p^4 + n\cdot r(4R+r)^3 \le (n+1)\cdot p^2(R+r)(4R+r), \text{ where } n \ge 0.$$

MARIN CHIRCIU, ROMANIA

200. Prove that in any triangle the following inequality is true:

$$n + \sin\frac{A}{2} \le \sqrt{\frac{(p-b+nc)(p-c+nb)}{bc}}, \text{ where } n \ge 0.$$

MARIN CHIRCIU, OCTAVIAN STROE, ROMANIA

201. Prove that if $a, b, c, d, e, f, \in (0,\infty)$ and $a+b+c=2$, $d+e+f=3$, then:

$$\left(\frac{d}{a}\right)^a \cdot \left(\frac{e}{b}\right)^b \cdot \left(\frac{f}{c}\right)^c \le \frac{9}{4}.$$

DANIEL SITARU, RMM, ROMANIA

202. If $x, y, z \in \left(0, \frac{\pi}{2}\right)$, then:

$$\frac{\tan x}{\sin y + \sin z} + \frac{\tan y}{\sin z + \sin x} + \frac{\tan z}{\sin x + \sin y} > \frac{3}{2}.$$

D.M. BĂTINEȚU-GIURGIU, NECULAI STANCIU, RMM, ROMANIA

203. If $n \in \mathbb{N}$, $n \geq 3$, $x, y \geq 0$, then:

$$\sqrt[3]{x^3 + y^3} + \sqrt[4]{x^4 + y^4} + \ldots + \sqrt[n]{x^n + y^n} \leq (n-2)\sqrt{x^2 + y^2}.$$

DANIEL SITARU, RMM, ROMANIA

204. If $a, b, c \geq 0$, then:

$$\frac{a + \sqrt{ab} + \sqrt[3]{abc}}{3} \leq \sqrt[3]{a\left(\frac{a+b}{2}\right)\left(\frac{a+b+c}{3}\right)}.$$

KIRAN KEDLAYA, RMM, USA

205. If $a, b, c, d > 0$, $x, y \in \mathbb{R}$, then:

$$\frac{\sin^2 x}{a} + \frac{\cos^2 x}{b} + \frac{\sin^2 y}{c} + \frac{\cos^2 y}{d} > \frac{2}{a+b+c+d}.$$

DANIEL SITARU, RMM, ROMANIA

206. If $a, b, c, d > 0$, then:

$$\left(2a^2\sqrt{b^3}\sqrt[3]{c^4}\sqrt[4]{d^5} + \frac{3}{2}b^2\sqrt{c^3}\sqrt[3]{d^4}\sqrt[4]{a^5} + \frac{4}{3}c^2\sqrt{d^3}\sqrt[3]{a^4}\sqrt[4]{b^5} + \frac{5}{4}d^2\sqrt{a^3}\sqrt[3]{b^4}\sqrt[4]{c^5}\right)\left(\sum\frac{1}{a}\right)^4 \geq$$

$$\geq \frac{4672}{3}.$$

UCHE ELIEZER OKEKE, RMM, NIGERIA

207. If $a, b, c \geq 0$, then:

$$a + b + c \geq \left(\sqrt[4]{a} + \sqrt[4]{b}\right)\sqrt[4]{abc}.$$

DANIEL SITARU, RMM, ROMANIA

208. If $0 \leq a < 3$, $0 \leq b < 5$, $0 \leq c < 7$, then:

$$\sqrt[3]{a+1} + \sqrt[5]{b+1} + \sqrt[7]{c+1} < 6.$$

DANIEL SITARU, RMM, ROMANIA

209. If $0 \leq a < b < c < d$, then:

$$3\sqrt{a} + 3\sqrt[4]{b} + 2\sqrt[6]{c} + \sqrt[8]{d} > \sqrt[6]{ab} + \sqrt[12]{abc} + \sqrt[20]{abcd}.$$

DANIEL SITARU, RMM, ROMANIA

210. If $x, y, z > 0$, $6xyz = \dfrac{1}{x+2y+3z}$, then:

$$\frac{(4x^2y^2+1)(36y^2z^2+1)(9z^2x^2+1)}{2304x^2y^2z^2} \geq \frac{1}{(x+2y+3z)^2}.$$

DANIEL SITARU, RMM, ROMANIA

211. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(x^2 + f(x)f(y)) = xf(x + y)$ for all real numbers x and y.

MEMO, 2017

212. In ΔABC, the following relationship holds:

$$\prod\left(m_a^2 + m_a m_b + m_b^2\right) \geq \left(\frac{9r_a r_b r_c}{r_a + r_b + r_c}\right)^3.$$

DANIEL SITARU, RMM, ROMANIA

213. In ΔABC, the following relationship holds:

$$\frac{b^4c^7}{a^{12}} + \frac{c^4a^7}{b^{12}} + \frac{a^4b^7}{c^{12}} \geq \frac{\sqrt{3}}{R}.$$

DANIEL SITARU, RMM, ROMANIA

214. In ΔABC, the following relationship holds:

$$\frac{h_b^5h_c^4}{h_a^{10}} + \frac{h_c^5h_a^4}{h_b^{10}} + \frac{h_a^5h_b^4}{h_c^{10}} \geq 3\sqrt[3]{\frac{R}{2S^2}}.$$

DANIEL SITARU, RMM, ROMANIA

215. In acute-angled ΔABC, the following relationship holds:

$$\left(\sum \frac{1}{\sqrt{s-a}}\right)^2 \leq (m_a + w_b + h_c) \cdot \frac{4R+r}{3sr^2}.$$

DANIEL SITARU, RMM, ROMANIA

216. In ΔABC, K – Lemoine's point. Prove that:

$$\frac{AK}{b^2+c^2} + \frac{BK}{c^2+a^2} + \frac{CK}{a^2+b^2} \leq \frac{m_a + m_b + m_c}{a^2+b^2+c^2}.$$

ADIL ABDULLAYEV, RMM, AZERBAIJAN

217. In ΔABC, the following equality holds:

$$\sum\left(\sqrt[3]{a} + \sqrt[3]{b} - \sqrt[3]{c}\right)^3 \geq \sqrt[3]{3a} + \sqrt[3]{3b} + 3\sqrt[3]{3c} - 2.$$

DANIEL SITARU, RMM, ROMANIA

218. Prove that in any triangle the double inequality is true:

$$\frac{6}{n+2}\cdot\frac{r}{R}\le\frac{a}{na+b+c}+\frac{b}{nb+c+a}+\frac{c}{nc+a+b}\le\frac{3}{2(n+2)}\cdot\frac{R}{r}, \text{ where } n\ge 0.$$

MARIN CHIRCIU, ROMANIA

219. Prove that in any triangle the following inequality is true:

$$p^4+n\cdot r(4R+r)^3\le(n+1)\cdot p^2(R+r)(4R+r), \text{ where } n\ge 0.$$

MARIN CHIRCIU, ROMANIA

220. Prove that in any ABC triangle the following inequality is true:

$$\sqrt{bc(p-na)}+\sqrt{ca(p-nb)}+\sqrt{ab(p-nc)}\le 3R\sqrt{(3-2n)p}, \text{ where } n\ge 1.$$

MARIN CHIRCIU, ROMANIA

221. Prove that in any ABC triangle the following double inequality is true:

$$a^2(b+c-na)+b^2(c+a-nb)+c^2(a+b-nc)\ge 24(2-n)\cdot\frac{S^2}{p}, \text{ where } n\le 1.$$

MARIN CHIRCIU, ROMANIA

222. Prove that in any ABC triangle the following inequality holds:

$$\frac{h_ah_b}{h_a+h_b}+\frac{h_bh_c}{h_b+h_c}+\frac{h_ch_a}{h_c+h_a}\ge\frac{9r}{2}.$$

MARIN CHIRCIU, ROMANIA

223. Prove that in any triangle the following inequality holds:

$$\left(\frac{4R+r}{p}\right)^2+\frac{nr}{4R+r}\ge 3+\frac{n}{9}, \text{ where } n\le 9.$$

MARIN CHIRCIU, ROMANIA

224. In acute-angled ΔABC, the following relationship holds:

$$\sum\frac{1}{\sin 2B+\sin 2C-\sin 2A}\ge\frac{3}{\sqrt[3]{\prod\sin 2A}}\ge 2\sqrt{3}.$$

DANIEL SITARU, RMM, ROMANIA

225. Let m_a, m_b, m_c be the lengths of the medians of a triangle, and let w_a, w_b, w_c be the lengths of the internal bisectors of the angle opposite of the sides of lengths a, b, c, respectively. Prove that: $\dfrac{\left(\dfrac{m_a^2}{a}+\dfrac{m_b^2}{b}+\dfrac{m_c^2}{c}\right)^2}{w_a^2+w_b^2+w_c^2}\ge\dfrac{9}{4}.$

GEORGE APOSTOLOPOULOS, RMM, GREECE

226. Let ABC be an arbitrary triangle, I_a, I_b, I_c are excenters, r is inradius, and R is circumradius of ABC. Prove that:

$$12r\sqrt{3} \leq P(I_aI_bI_c) \leq 6R\sqrt{3},\ \text{where } P(I_aI_bI_c) \text{ is perimeter of } ABC.$$

MEHMET ȘAHIN, RMM, TURKEY

227. Let a, b, and c be the side lengths of a triangle ABC, with inradius r. Prove that:

$$\sqrt[4]{\frac{a^4}{\tan^2\frac{A}{2}}+\frac{b^4}{\tan^2\frac{B}{2}}+\frac{c^4}{\tan^2\frac{C}{2}}} \geq 6\cdot r.$$

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228. In any triangle ABC, the following relationship holds:

$$\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{C}{2} \leq \frac{R}{2\cdot r}\sqrt{\frac{2R}{r}-1}.$$

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229. In ΔABC, the following relationship holds:

$$2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right) \leq \frac{3}{\sqrt[3]{(b+c-a)(c+a-b)(a+b-c)}}.$$

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230. In ΔABC, the following relationship holds:

$$a^6+b^6+c^6 \geq 8r^2s\sum\frac{a^5}{b^2-bc+c^2}.$$

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231. Prove that in any triangle ABC:

$$\frac{m_a}{l_a}+\frac{m_a}{l_b}+\frac{m_c}{l_c} \geq \frac{\sqrt{b}+\sqrt{c}}{2\sqrt{a}}+\frac{\sqrt{c}+\sqrt{a}}{2\sqrt{b}}+\frac{\sqrt{a}+\sqrt{b}}{2\sqrt{c}},$$

where l_a, l_b, l_c are internal angle bisectors from A, B, C, respectively.

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232. Prove that in any ABC triangle the following inequality holds:

$$\frac{\sin A}{m\sin B+n\sqrt{\sin A\sin B}}+\frac{\sin B}{m\sin C+n\sqrt{\sin B\sin C}}+\frac{\sin C}{m\sin A+n\sqrt{\sin C\sin A}} \geq \frac{3}{m+n},$$

where $2m \geq n > 0$.

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